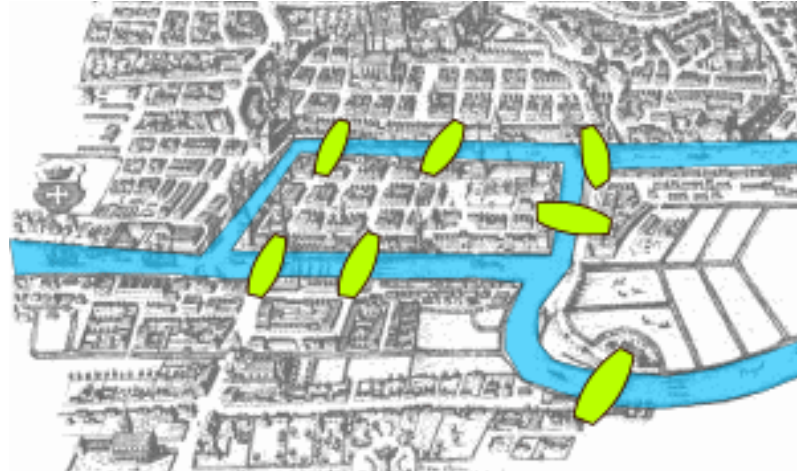


Graph Theory

MATH 3220
By
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Seven Bridges of Königsberg

- The first known work on graph theory was Leonhard Euler's paper on The Seven Bridges of Königsberg in 1736.
- The problem of the seven bridges was to traverse each bridge of Königsberg once and only once.



Source: Wikipedia

Seven Bridges of Königsberg (cont)

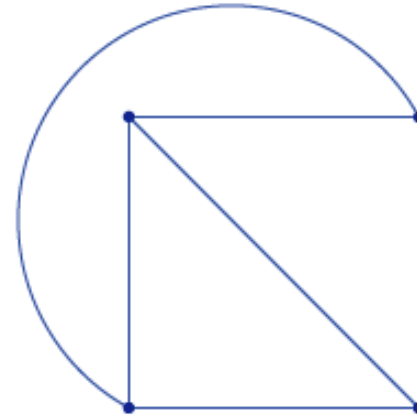
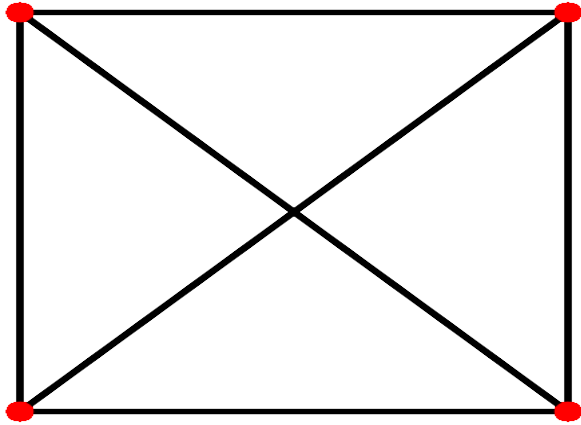
- Euler concluded that this was impossible and realized that only the number of vertices and paths were relevant in the problem.
- Because only the vertices and lines were important, the diagram for a graph became combinations of points and the lines connecting them.

Basic Graphs

A graph consists of two sets named the vertex set and the edge set. The vertex set is a nonempty set, and the edge set may be empty, but if not it contains two-element subsets of the vertex set.

In drawing the graph, the location of the dots and lines are irrelevant, but how they connect to each other does matter. Also, the direction of the lines may be added resulting in a directed graph.

Drawing Graphs

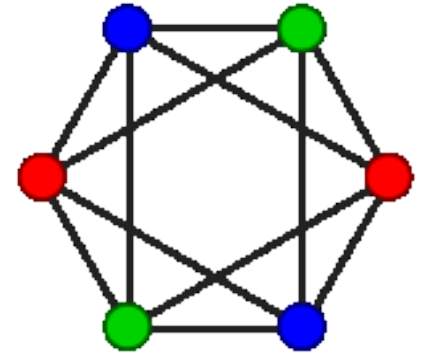


- The two above diagrams are equal graphs.
- They are also complete graphs of vertex 4.
- A complete graph has all possible edges of a positive integer of vertices.

Source: Wikipedia, Britannica

Graph Coloring

- Graph coloring involves labeling each vertex of a graph with a color so that all adjacent vertices have different colors.
- Another part of graph coloring involves edge coloring. In it, each edge of a graph is colored so that every adjacent edge has a different color.
- The smallest possible number of colors that works for this is called the graph's chromatic number.



Planer Graphs

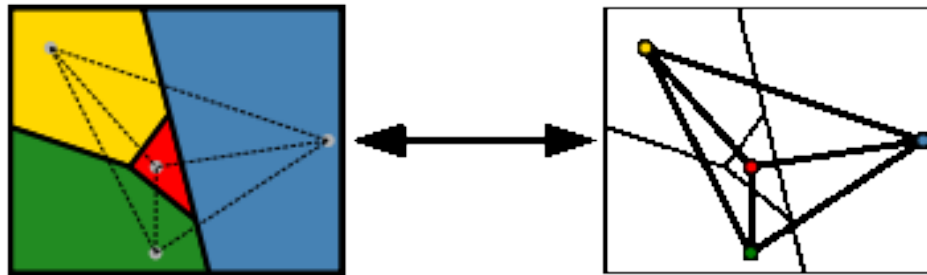
- A planer graph is a graph that can be drawn in a plane without intersections.
- When these graphs are drawn without edge-crossings, the regions that the edges form in the graph are called faces.
- Using these concepts, Euler proved in 1752 that the number of vertices and faces minus the number of edges is always 2.
- Put into terms: $v + f - e = 2$

Four Color Theorem

- The four color theorem says that every vertex in a planar graph can be colored with just four different colors.
- The theorem was first introduced in 1852 by Francis Guthrie.
- The final proven theorem was solved in 1976 by Appel and Haken, and their proof was checked by a computer.

Four Color Theorem (cont)

- This problem is generally drawn as a map such as a political or boundary map.
- By using each country as a vertex, the problem becomes a vertex coloring problem.



Source: Wikipedia

Other Subjects in Graph Theory

- Platonic Graphs
- Hamilton Graphs
- Directed Graphs or Digraphs
- Networks
- Random Graph Theory
- Extremal Graph Theory
- Weighted Graphs
- Trees

Open Problems in Graph Theory

- The Erdős–Faber–Lovász conjecture (unsolved)
- The total coloring conjecture (unsolved)
- The list coloring conjecture (unsolved)
- Traveling Salesman Problem
- Chinese Postman Problem
- Three Cottage Problem

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